

An Algorithm for Forecasting Earthquake Ground Motion

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A first version of an algorithm is described designed to estimate expected strong ground motion due to earthquakes for use in civil engineering seismology. The algorithm has the following features: (1) the properties of seismic events in a seismic source zone are specified in either of the following three ways: first, by using the Fourier acceleration spectrum (or else the response spectrum) of a typical (reference) earthquake which has been recorded or is expected at a reference distance from the event of a reference magnitude; secondly, by specifying a regional model of Fourier acceleration spectrum in tabular form (for a set of magnitudes at a reference distance); and thirdly, by using an analytical spectral model; (2) spectral levels and other parameters at arbitrary distances and magnitudes are found by making use of appropriate scaling laws for earthquake source parameters and spectral levels, as well as of attenuation and scattering parameters; (3) site conditions are incorporated through a set of corrections to the Fourier spectrum for soils of categories 2 and 3 relative to category 1. The block diagram of the algorithm is presented with the description of its modules. Examples are given to illustrate the calculations of the Fourier spectrum, response spectrum, peak acceleration, and duration as functions of the magnitude and/or the distance.

INTRODUCTION

Forecasting the parameters of ground motion with specified earthquake source parameters, wave path, and construction site is a very important problem in the general problem of seismicity mapping. A correct solution is impeded, among other things, by discrepancies between the available empirical regression relations for various ground motion parameters and contradictions between their structure and the simple ground motion theory. The situation is frequently aggravated by a partial or complete absence of observations in poorly studied regions or in low seismicity areas. This paper describes the first practical version of algorithm designed to compute ground motion that may take place as a result of a potential earthquake in a given area. Ideally, such a forecast should incorporate

detailed source and path effects, as well as site conditions. However, the procedure is usually simplified in engineering seismology by describing the earthquake source through the magnitude, the path through the hypocentral distance, and the site through one of three soil categories. Here we describe a version of this simplified approach, distinguished by the following features: (1) the properties of events in a seismic source zone are described by the Fourier spectrum FS of horizontal accelerations excited by a hypothetical or actual typical (reference) earthquake of a reference magnitude M_{w0} recorded on bedrock at a reference distance r_0 , (2) the calculations of r and M_w values are performed by using scaling laws for source parameters and spectra and specifying attenuation and scattering in the medium; (3) site conditions are incorporated by using amplitude-independent corrections to the spectrum for the soil category, the effect of ground conditions on the duration being disregarded.

The algorithm is versatile: the reference Fourier spectrum can be specified on the basis of the Brune model or calculated from the reference response spectrum. The algorithm yields all main ground motion parameters: peak velocity, maximum acceleration, duration, Fourier spectra, and response spectra, power spectra.

GENERAL DESCRIPTION OF THE ALGORITHM AND ITS MODULES

The block diagram in Fig. 1 illustrates the principal relations between the input, intermediate, and output quantities: M_w and r for a specific earthquake source, path and soil conditions, signal characteristics at the source and at distances r_0 and r , and ground motion parameters. To adjust the algorithm for a specific region or epicentral zone it was advisable to use, as a major input characteristic, the parameters that describe the Fourier acceleration spectrum FS for specific fixed value of magnitude $M_w = M_{w0}$ and distance $r = r_0$. These quantities are assumed to be known from observations in a region of study or in other similar regions, or else can be found by informal data generalization or calculated using an analytical model. The Fourier spectrum can also be computed from a response spectrum for fixed M_{w0} and r_0 values.

Further, corrections to FS for attenuation and geometrical spreading are calculated for the whole length of the source using on given input values of M_w and distance to the center of the source r . Corrections to the FS level are computed using a difference between M_w and M_{w0} and a difference of the soil conditions from bedrock. The following parameters are used to characterize the medium: c_s , shear velocity; Q_0 , quality factor at $f = 1$ Hz; γ_Q , exponent in the dependence of Q on frequency. Soil conditions are characterized by the parameter g - a soil category.

Rupture duration and source length are important quantities for the calculation. They are estimated from the magnitude based on the similarity of earthquake sources. Earlier [5] a specific relation was derived for this purpose through the generalization of observational data. The parameters of the reference Fourier spectrum and the corrections are used to compute the main ground motion characteristic for fixed M_w and r - the Fourier acceleration spectrum.

The second main characteristic, accelerogram duration, was found from precomputed

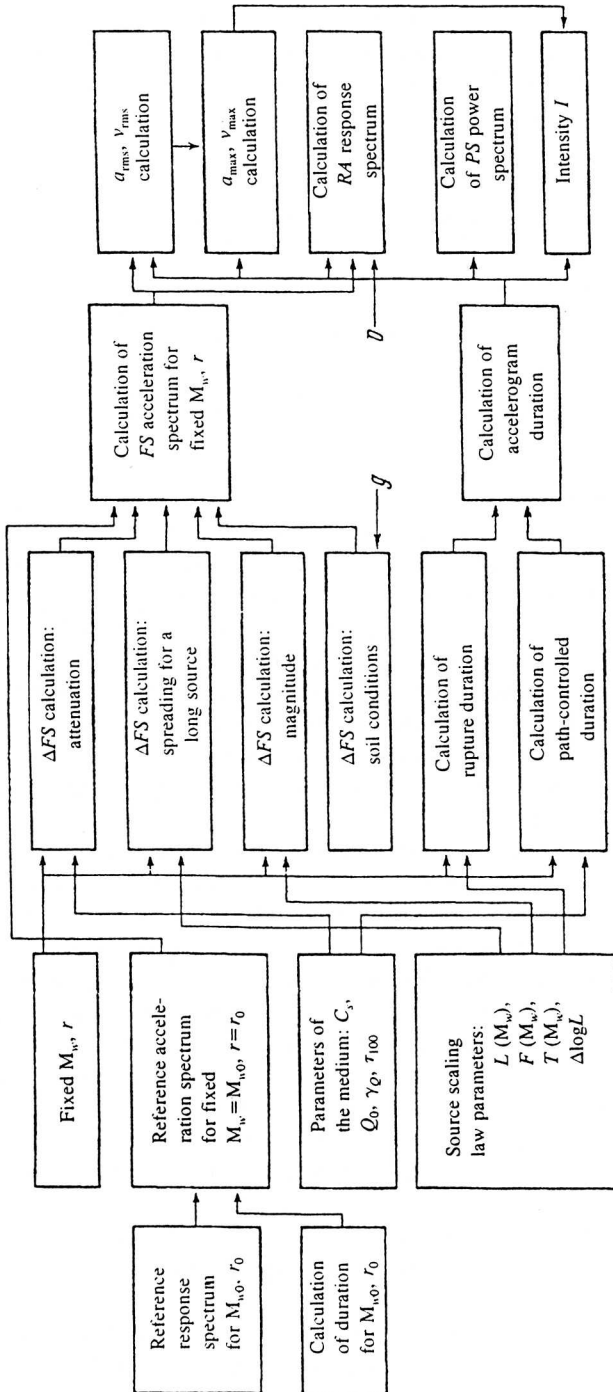


Figure 1 Block diagram of the algorithm.

rupture duration and the duration component determined by the medium. The path effect was incorporated using the parameter τ_{100} which describes the duration of an S -wave packet excited by an instantaneous source (impulsive response of the medium) at hypocentral distance 100 km. Given a Fourier spectrum and duration, the other ground motion parameters can be found.

Table 1 Fourier spectrum: corrections for soil conditions.

Soil category	f , Hz								
	0.20	0.32	0.5	1	2	3.2	5	10	20
2	0.15	0.22	0.26	0.29	0.23	0.18	0.10	0	-0.10
3	0.27	0.40	0.48	0.55	0.43	0.27	0.11	-0.10	-0.30

Basic data. *Specifying the basic reference spectrum.* In the main variant, the reference spectrum is given in tabular form as an actually observed spectrum based on observations in the study area or as a forecast based on the generalization of various observations. The reference spectrum can be calculated from a response spectrum given in tabular form. It is also possible to use for the reference spectrum a regional spectral model given in tabular form for a set of M_w values at distance r_0 .

An alternative version of the algorithm uses an analytical representation of the source spectrum. The Brune model modified by Boore and Joyner [7] is adjusted using seismic moment M_0 , stress drop $\Delta\sigma$, and corner frequency f_{\max} .

Magnitude conversion can be done from M_w to M_{LH} using an appropriate regression given in tabular form.

Specifying the properties of the medium. The medium is assumed to be a homogeneous, poorly attenuating, elastic half space. It is assumed to scatter energy in the manner that no energy is lost (forward scattering), the role of scattering being that the duration of the record from an instantaneous source increases with distance. As a matter of fact, the effects of forward scattering and dispersion are indistinguishable at frequencies ~ 1 Hz; the software does not incorporate the combined effect on an empirical basis.

Specifying the scaling law for spectra and source parameters. The spectra and source parameter scaling is a very important procedure in the main variant for calculating corrections and the total duration. The scaling law was derived as a result of generalizing observational data and using the hypothesis of source similarity. The source size and rupture duration were assumed to grow according to a power law as functions of the moment M_0 .

Assuming strict similarity, we calculated source length L (following [5]) from

$$\log L = 0.5 M_w - 1.85 + a. \quad (1)$$

Here, $a = \log L$ is a correction related to a possible deviation of the stress drop $\Delta\sigma$ from the mean value. Relation (1) was used to compute rupture duration assuming that

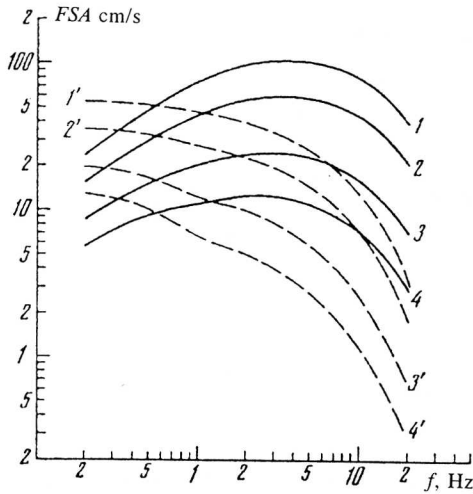


Figure 2 Fourier acceleration spectrum $FSA(f)$ for $M_w = 8$ and $r = 25, 50, 100,$ and 150 km (1, 1' to 4, 4', respectively). The reference spectra are the spectrum recommended for Kamchatka [5] (1-4) and the Brune analytical model (1'- 4').

$$L = T_{src} v. \tag{2}$$

We used $v = 3.5$ km/s following [5].

The variation of the Fourier short period acceleration spectrum with magnitude M_w was specified on the bases of observed $FS(M_w)$ data. These data were available for the near zone and for teleseismic distances. Although the near-zone data are in principle preferable, their reliability for moment magnitudes of 7 to 9 is low even in well studied regions, such as California and Japan. For this reason the spectral shape was assumed to be independent of magnitude, and the spectral trend was derived from teleseismic data [10] as

$$\log(FS) = 0.6M_w + const. \tag{3}$$

Note that the Brune spectral model predicts a slower growth at high frequencies (with a slope of 0.5).

This component of the algorithm is still experimental.

Calculation of acceleration spectrum. Attenuation. Inelastic attenuation is incorporated through the introduction of a correction factor into the spectrum:

$$K_Q(f) = \exp \left[-\frac{\pi f r}{Q(f) c_s} \right], \tag{4}$$

where the frequency-dependent Q -factor is given as

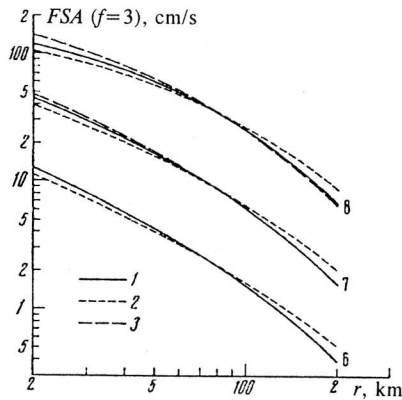


Figure 3 Fourier acceleration spectrum $FSA (f=3)$ as a function of distance r : 1 - based on the basic set of parameters, 2 - for $Q_0 = 270$, 3 - for $\Delta \log L = -0.17$. Numbers on the right of the curves are magnitudes M_w .

$$Q(f) = \begin{cases} Q_0 & \text{for } f < f_0 = 1 \text{ Hz,} \\ Q_0 f^{\gamma Q} & \text{for } f \geq f_0 = 1 \text{ Hz.} \end{cases} \quad (5)$$

Incorporation of spreading for a long source. The Fourier spectrum is lower near a long source than near a point source, because the effective radiating area decreases as the recording point moves closer to the source. Trifunac and Lee [11] reported a comparative study of four relations that describe the radiation field near a long source:

- a simple relation that describes the usual saturation of the Fourier spectrum near the source:

$$K_r = (R_{\text{eff}}^2 + r^2)^{-1/2}, \quad (6)$$

and corresponds to an illumination engineering model - a circular area that radiates according to the Lambert law;

- Gusev's relation [3]:

$$K_r = \left[\frac{1}{R_{\text{eff}}^2} \ln \left[\frac{r^2 + R_{\text{eff}}^2}{r^2 + R_c^2} \right] \right]^{1/2} \quad (7)$$

(here R_c is the coherence radius of the source, taken to 1 km), which is different from formula (6) in that a more realistic isotropic radiation pattern of an elementary radiator has been used

- and two modified versions of (6) and (7).

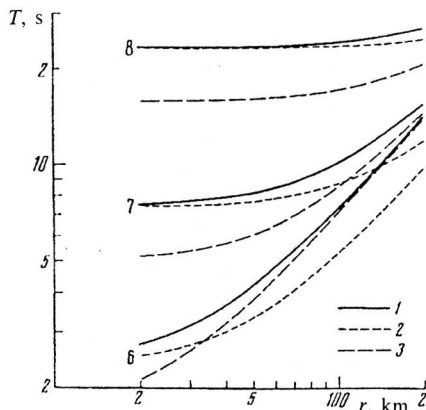


Figure 4 Effective duration T_{eff} as a function of distance: 1 - based on the basic parameter set, 2 - for $\tau_{100} = 2.4$ s, 3 - for $\Delta \log L = -0.17$.

Fitting these relations to the California data base revealed that formula (7) showed the best fit. It was thus recommended for California. Because this study was based on the world-best regional data base, we have used formula (7) in our algorithm.

Following [3], the effective source radius (in terms of radiation intensity) R_{eff} was estimated from

$$R_{\text{eff}} = \frac{1}{2} (LW)^{1/2} \approx 0.4L. \tag{8}$$

This result was used in formula (7).

Another, more accurate variant of computing finite source size correction uses the Gusev-Shumilina program for computing the radiation field due to a rectangular source. The resulting source shape is more realistic.

Incorporation of magnitude. To incorporate the effect of the magnitude on the spectral level, equation (3) was used to derive

$$FS(M_w) = K_m FS(M_{w0}), \tag{9}$$

where

$$\log K_m = 0.6 (M_w - M_{w0}). \tag{10}$$

For the frequency range 0.5-10 Hz, expression (3) is valid within the magnitude range $M_w = 6.5 \dots 9$. The parameters in (3) can be adjusted to fit a particular region.

An alternative variant incorporates the magnitude automatically, because the model is analytical.

Incorporation of soil conditions. Soil conditions are incorporated by using corrections

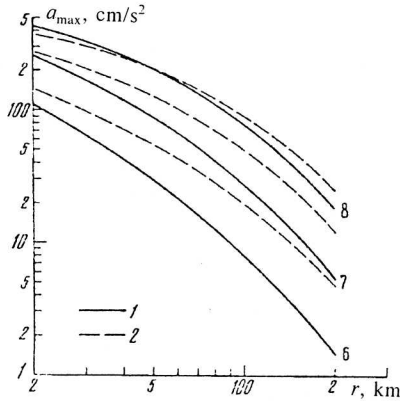


Figure 5 Calculation of a_{\max} as a function of r for the magnitudes $M_w = 6, 7,$ and 8 (1). Shown for comparison are the values of a_{\max} based on data from [9] for the same magnitudes (2).

to $\log FS(r, M_w)$ given in tabular form. According to the current classification, soil conditions are classified into bedrock, intermediate, and soft soil. Corrections are assumed to be independent of soil thickness and the level of earthquake excitation. Ground motion duration is assumed to be equal to that in bedrock. As an example, Table 1 presents a variant of corrections for soil conditions computed and recommended for Kamchatka in [4] in order to take account of amplification in the low impedance layer and of nonlinear effects during strong ($a_{\max} = 0.1 \dots 0.2 g$) ground motion.

Calculation of the Fourier spectrum. The Fourier spectrum for fixed M_w and r are computed by either of the following three procedures: (1) based on reference Fourier spectrum for fixed $M_w = M_{w0}$ and $r = r_0$; (2) based on a regional of $FS(f, M_w)$ model for fixed r_0 ; (3) based on the source spectrum derived from the analytical model.

In variant 1, the Fourier spectrum for fixed r and M_w values is given by

$$FS(f|r, M_w) = FS(f|r_0, M_{w0}) K_m K_Q(f) K_r K_g, \quad (11)$$

where K_m , K_Q , K_r , and K_g are correction factors incorporating magnitude, attenuation, spreading, and soil conditions in accordance with (10), (4), (7), and Table 1, respectively. To illustrate this computation procedure, the program uses a mean spectrum for large Kamchatkan earthquakes obtained in [4, Fig. 4] for $M_{w0} = 8.4$ and $r_0 = 80$ km.

In variants 2 and 3, the Fourier spectrum level is determined from a similar relation where $FS(f|r_0, M_{w0})$ is found by the interpolation of tabular values or calculated analytically, respectively.

Calculation of duration. The source pulse is elongated away from the fault as a result of scattering and dispersion. Simple theory shows that scattering of an incoherent pulse can be described as a convolution of its squared envelope with the square of the envelope

of the impulsive response of the medium (with Green's power function). The square of the rms duration of the convolution is equal to the sum of squares of the rms durations of the envelopes. The use of this helpful fact required the introduction of a new parameter, T_{rms} . This exact result for incoherent narrow-band quasistationary signals can be used as an approximation for the entire band of 0.5-10 Hz.

Calculation of rupture duration. Taking a model signal with a rectangular envelope and total duration T_m , the rms duration T_{rms} is related to T_m as

$$T_{\text{rms}} = T_m / \sqrt{12} \quad (12)$$

(this is the well-known expression for the variance of the rectangular law in probability theory). The assumption of $T_m = T_{\text{src}}$ in (2) and the use of (12) gives T_{rms} for the source, which is denoted here as $T_{\text{src.rms}}$.

Calculation of the path-controlled duration component. The rms duration of the impulsive response of the medium is estimated from the empirical relation

$$T_{\text{path}} = \tau_{100} (r^{n_m} / 100), \quad (13)$$

where the exponent n_m is taken to be equal to unity, and τ_{100} (the duration for $r=100$ km) is an adjustable input parameter.

Calculation of accelerogram duration. The total rms duration is found from

$$T_{\text{rms}} = (T_{\text{src.rms}}^2 + T_{\text{path}}^2)^{1/2}. \quad (14)$$

When calculating peak acceleration and response spectrum in the interval of maximum accelerogram amplitudes, it is convenient to use effective duration T_{eff} . This is the duration of a signal of the same energy having a rectangular envelope and an rms amplitude equal to the rms amplitude of the input signal around the maximum:

$$T_{\text{eff}} \approx 2 T_{\text{rms}}. \quad (15)$$

The factor 2 in (15) may require some modification.

Calculation of ground motion parameters. *Calculation of power spectrum.* The power spectrum $PS(f)$ is found from the simple relation

$$PS(f) = FS^2(f) / T_{\text{eff}}. \quad (16)$$

Calculation of a_{rms} and v_{rms} . The rms acceleration is evaluated from the Parseval equation for signal energy in the frequency and time domains:

$$a_{\text{rms}}^2 T_{\text{eff}} = 2 \int_0^{\infty} FS^2(f) df. \quad (17)$$

The rms velocity v_{rms} is found from a similar relation in which $FS(f)$ is replaced as follows:

$$FSV(f) = FS(f) / 2\pi f. \quad (18)$$

Calculation of a_{max} and v_{max} . The excess of the maximum peak in acceleration a_{max} over the rms acceleration is evaluated from the formula valid for a segment of the Gaussian process [2]:

$$a_{\max}^2 = (\ln(n) + 0.577) a_{\text{rmsp}}^2, \quad (19)$$

where $n \approx 2\hat{f}T_{\text{eff}}$ is the number of peaks, a_{rmsp} is the rms peak. The mean frequency, \hat{f} , is found from

$$\hat{f} = \frac{\int_0^{\infty} f FS^2 df}{\int_0^{\infty} FS^2 df}. \quad (20)$$

To evaluate a_{rmsp} we assume $a_{\text{rmsp}}^2 = 2a_{\text{rms}}^2$ (this is an exact result for mathematical expectations in a broad class of random processes). The final relation is

$$a_{\max} = a_{\text{rms}} [2 (\ln(2\hat{f}T_{\text{eff}}) + 0.577)]^{1/2}. \quad (21)$$

The Gaussian law for accelerograms is here assumed by tradition and possibly needs modification.

The algorithm for calculating peak velocity is only different from that for acceleration in that $FS(f)$ is replaced by $FSV(f)$ in accordance with relation (18)).

Calculation of acceleration response spectrum RA. The response spectrum RA is estimated from Fourier spectrum FS using the following compact set of relations derived in [4]

$$\begin{aligned} RA(f) &= 2\pi f RV(f), \\ RV(f) &= C_v |FS(f)|, \\ C_v &= \frac{A(q)}{2q} (1 - e^{-2q}), \\ A(q) &= \begin{cases} 1 & \text{for } q \leq 1, \\ \sum_{i=1}^n 1/i & \text{for } 1 < q < 16, \\ \ln(n) + 0.577 & \text{for } q \geq 16, \end{cases} \end{aligned} \quad (22)$$

where $q = T_{\text{eff}}/T_{\text{osc}}$; $T_{\text{osc}} = 1/(2\pi fD)$ is the characteristic time of a transient process in an oscillator having natural frequency f and damping D ; $n = (q - 1)/\pi$ is the number of independent peaks.

Note that the above relations are valid for $D \ll 1$. A typical value, and that actually used in the calculation, was $D = 0.05$.

Calculation of expected intensity. Seismic intensity is found from the Aptikaev relation [1]

$$I = 3.3 (\log a_{\max} + 0.44 \log T_{\text{eff}}) + C. \quad (23)$$

The constant C is taken to be equal to -0.45 in order to make the two following intensity estimates identical: one found from (23) with the a_{\max} and T_{eff} values obtained for $M_w = 6$ and $r = 50$ km; the other found from Shebalin's relation [6] with $M_{LH} = 6.0$ (corresponding to $M_w = 6$) and $r = 50$ km.

Accurate fitting to achieve consistency between I and ground motion parameters is done when adjusting the model for a specific region.

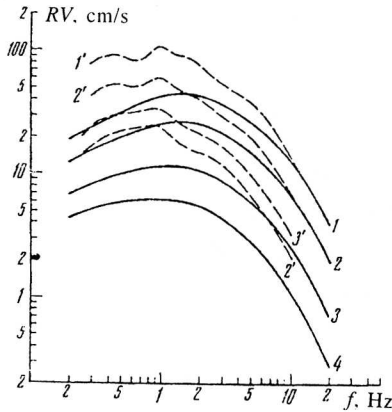


Figure 6 Velocity response spectrum $RV(f)$ for $M_w = 8$, $r = 25, 50, 100,$ and 150 km (1 to 4, respectively), and intermediate soil (1 to 4). Plotted for comparison are the response spectra $RV(f)$ based on data from [8] for the same M_w and r (1' to 4').

EXAMPLES

The procedure is illustrated by presenting calculations for the relations $FSA(f)$, $FSA(r)$, $T_{\text{eff}}(r)$, $a_{\text{max}}(r)$, $RV(f)$, and $RA(f)$. The first five relations were calculated using the reference spectrum recommended for Kamchatka in [4] at $M_{w0} = 8.4$ and $r_0 = 80$ km. The calculation performed was for the distance range between 20-200 km and magnitudes $M_w = 6, 7,$ and 8 with the following basic set of parameter values: $Q_0 = 180$, $\gamma_Q = 0.75$, $C_s = 3.5$ km/s, $\tau_{100} = 3.5$ s, $\Delta \log L = 0$. Soil conditions were assumed to bedrock, unless specified otherwise.

Fourier spectrum calculation. Figure 2 presents examples of the Fourier acceleration spectra calculation in the frequency range 0.2-20 Hz for $M_w = 8$ and $r = 25, 50, 100,$ and 150 km. The reference spectra were the spectrum recommended for Kamchatka [4] and the Brune analytical spectral model [7]. The Brune model calculations were based on values typical of subduction zones: stress drop $\Delta \sigma = 50$ bars, $C_s = 3.5$ km/s, and corner frequency $f_{\text{max}} = 10$ Hz. The Brune spectrum level at high frequencies is noticeably below the Kamchatka spectrum. This is a well-known feature of the Brune model which disregards the difference between the global and the local stress drop.

$FSA(r)$ and $T_{\text{eff}}(r)$ relations. Variations of the main parameters. The influence of the model parameters on the major characteristics of the accelerogram (Fourier spectrum and duration) is illustrated by calculating the $FSA(r)$ and $T_{\text{eff}}(r)$ relations for the basic parameter set and for Q_0 , τ_{100} , and $\Delta \log L$ modified by a factor of 1.5. The $FSA(r)$ relation was calculated around the spectral maximum at $f = 3$ Hz. The results are shown in Figs. 3 and 4. The characteristic features of the curves are as follows. First, the depen-

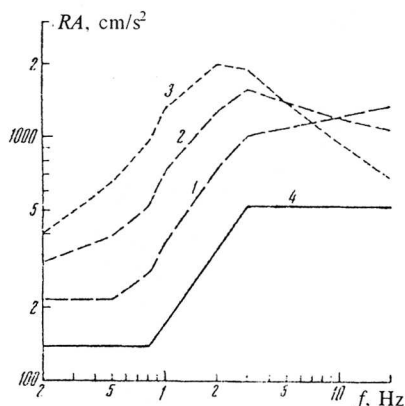


Figure 7 Acceleration response spectrum $RA(f)$ for soil categories 1, 2, and 3 $M_w = 7.4$ and $r = 30$ km (1 to 3, respectively); 4 - reference response spectrum according to SNiP II-7-81 for $M_w = 8$ and $r = 100$ km.

dence on distance has a saturation region near the source (near zone), its typical size being the size of the source. Secondly, the dependence of FSA (as well as of a_{max}) on magnitude M_w is not as strong near the source as in the far zone, corresponding to the hypothesis of a stable energy flux per unit source area. Thirdly, duration in the far zone steadily increases due to scattering. It should be remembered when examining curves 2 that the spectrum level at distance r_0 is fixed, so the curves have a common point at $r_0 = 80$ km when Q_0 and $\Delta \log L$ are varied.

$a_{max}(r)$ relation. This relation was calculated for the magnitudes $M_w = 6, 7,$ and 8 . The results are presented in Fig. 5. The following correspondence valid for Kamchatka should be borne in mind when examining the curves: ($M_w = 8$) \rightarrow ($M_{LH} = 7.9$); ($M_w = 7$) \rightarrow ($M_{LH} = 7.3$); ($M_w = 6$) \rightarrow ($M_{LH} = 5.9$). Shown for comparison are the $a_{max}(r)$ relations derived in [9] from observations in Japan for the same magnitudes M_w .

$RV(f)$ calculation. The velocity response spectrum $RV(f)$ was calculated for $M_w = 8$, $r = 25, 100,$ and 150 km, and intermediate soil. The results are presented in Fig. 6. Shown for comparison are the values obtained the same magnitude and distances using empirical relations derived for Japan (northern Honshu) in [8].

$RA(f)$ calculation. Figure 7 shows the acceleration response spectra calculated with a response spectrum taken to be a reference spectrum. The response spectrum used was taken from SNiP II-7-81 (Construction Norms and Specifications) for $a_{max} = 175$ cm/s² (this approximately corresponds to $M_{w0} = 8$, $r = 100$ km, and bedrock). Plotted in the figure are the original spectrum and the response spectra calculated for all three soil categories with $M_w = 7.4$ and $r = 30$ km.

When calculating the acceleration response spectra (Fig. 8), the velocity response spec-

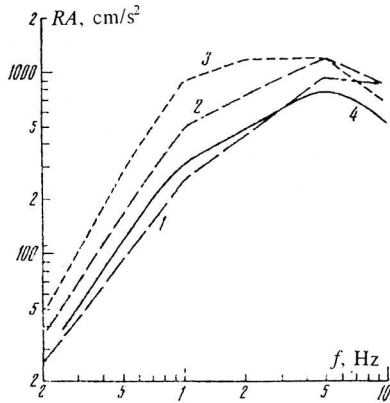


Figure 8 Acceleration response spectrum $RA(f)$ for $M_w = 7.4$ and $r = 30$ km: 1, 2, and 3 are soil categories 1 to 3, respectively; 4 - reference response spectrum from [8] for the same M_w and r .

trum from [8] was used as a reference. The figure shows response spectra for all three soil categories, $M_w = 7.4$, and $r = 30$ km, as well as the response spectrum which was computed using data from [8] for the same magnitude and distance.

CONCLUSION

This algorithm and the program based on the idea of relation (3) represent a new technique for a joint prediction of all strong ground motion parameters. The technique has a number of substantial advantages over empirical relations: it explicitly involves a number of complicated nonlinear interrelations between ground motion parameters and the properties of the source and the medium, and provides internally consistent results for the entire parameter set. Practical application in a particular region requires relevant adjustment of the program.

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